

First Ten Moments of Metalog Distributions with up to Five Terms

This document provides exact expressions for the first ten moments of metalog probability distributions with any number of terms from 2 through 5. These moments were calculated and validated with the aid of Wolfram Mathematica by Tom Keelin, August, 2016.

The 5 term metalog quantile function (where y is cumulative probability, $0 < y < 1$) is:

$$M_5(Y) := \left(a_1 + \text{Log} \left[\frac{Y}{1-Y} \right] a_2 + \left(-\frac{1}{2} + Y \right) \text{Log} \left[\frac{Y}{1-Y} \right] a_3 + \left(-\frac{1}{2} + Y \right) a_4 + \left(-\frac{1}{2} + Y \right)^2 a_5 \right)$$

moments notation

$\mu'_{1,5}$ mean of the 5 term metalog

$\mu_{k,5}$ kth central moment of the 5 term metalog

(For moments of the j -term metalog, where $j < 5$, set $a_i = 0$ for all $i > j$.)

first four moments of 5-term metalog

$$\mu'_{1,5} := a_1 + \frac{a_3}{2} + \frac{a_5}{12}$$

$$\mu_{2,5} := \frac{1}{3} \pi^2 a_2^2 + \frac{a_3^2}{12} + \frac{1}{36} \pi^2 a_3^2 + a_2 a_4 + \frac{a_4^2}{12} + \frac{a_3 a_5}{12} + \frac{a_5^2}{180}$$

$$\mu_{3,5} := \pi^2 a_2^2 a_3 + \frac{1}{24} \pi^2 a_3^3 + \frac{1}{2} a_2 a_3 a_4 + \frac{1}{6} \pi^2 a_2 a_3 a_4 + \frac{1}{8} a_3 a_4^2 + a_2^2 a_5 + \frac{1}{24} a_3^2 a_5 + \frac{1}{180} \pi^2 a_3^2 a_5 +$$

$$\frac{1}{4} a_2 a_4 a_5 + \frac{1}{60} a_4^2 a_5 + \frac{1}{120} a_3 a_5^2 + \frac{a_5^3}{3780}$$

$$\mu_{4,5} := \frac{7}{15} \pi^4 a_2^4 + \frac{3}{2} \pi^2 a_2^2 a_3^2 + \frac{7}{30} \pi^4 a_2^2 a_3^2 + \frac{a_3^4}{80} + \frac{1}{24} \pi^2 a_3^4 + \frac{7 \pi^4 a_3^4}{1200} + 2 \pi^2 a_2^3 a_4 + \frac{1}{2} a_2 a_3^2 a_4 +$$

$$\frac{2}{3} \pi^2 a_2 a_3^2 a_4 + 2 a_2^2 a_4^2 + \frac{1}{6} \pi^2 a_2^2 a_4^2 + \frac{1}{8} a_3^2 a_4^2 + \frac{1}{40} \pi^2 a_3^2 a_4^2 + \frac{1}{3} a_2 a_4^3 + \frac{a_4^4}{80} + a_2^2 a_3 a_5 +$$

$$\frac{1}{2} \pi^2 a_2^2 a_3 a_5 + \frac{1}{24} a_3^3 a_5 + \frac{1}{40} \pi^2 a_3^3 a_5 + \frac{5}{6} a_2 a_3 a_4 a_5 + \frac{2}{45} \pi^2 a_2 a_3 a_4 a_5 + \frac{3}{40} a_3 a_4^2 a_5 +$$

$$\frac{1}{6} a_2^2 a_5^2 + \frac{1}{90} \pi^2 a_2^2 a_5^2 + \frac{1}{45} a_3^2 a_5^2 + \frac{11 \pi^2 a_3^2 a_5^2}{7560} + \frac{1}{15} a_2 a_4 a_5^2 + \frac{11 a_4^2 a_5^2}{2520} + \frac{1}{420} a_3 a_5^3 + \frac{a_5^4}{15120}$$

5th central moment of 5-term metalog (49 terms)

$$\begin{aligned}
 \mu_{5,5} := & \frac{14}{3} \pi^4 a_2^4 a_3 + \frac{5}{3} \pi^2 a_2^2 a_3^3 + \frac{49}{36} \pi^4 a_2^2 a_3^3 + \frac{5}{144} \pi^2 a_3^5 + \frac{49 \pi^4 a_3^5}{2160} + \frac{25}{3} \pi^2 a_2^3 a_3 a_4 + \\
 & \frac{7}{9} \pi^4 a_2^3 a_3 a_4 + \frac{1}{4} a_2 a_3^3 a_4 + \frac{5}{4} \pi^2 a_2 a_3^3 a_4 + \frac{7}{60} \pi^4 a_2 a_3^3 a_4 + \frac{5}{2} a_2^2 a_3 a_4^2 + \frac{25}{12} \pi^2 a_2^2 a_3 a_4^2 + \\
 & \frac{5}{48} a_3^3 a_4^2 + \frac{7}{72} \pi^2 a_3^3 a_4^2 + \frac{5}{6} a_2 a_3 a_4^3 + \frac{1}{12} \pi^2 a_2 a_3 a_4^3 + \frac{7}{144} a_3 a_4^4 + \frac{10}{3} \pi^2 a_2^4 a_5 + a_2^2 a_3^2 a_5 + \\
 & \frac{25}{12} \pi^2 a_2^2 a_3^2 a_5 + \frac{7}{90} \pi^4 a_2^2 a_3^2 a_5 + \frac{1}{48} a_3^4 a_5 + \frac{7}{144} \pi^2 a_3^4 a_5 + \frac{1}{360} \pi^4 a_3^4 a_5 + 5 a_2^3 a_4 a_5 + \\
 & \frac{5}{6} \pi^2 a_2^3 a_4 a_5 + \frac{25}{24} a_2 a_3^2 a_4 a_5 + \frac{31}{72} \pi^2 a_2 a_3^2 a_4 a_5 + \frac{5}{3} a_2^2 a_4 a_5 + \frac{1}{18} \pi^2 a_2^2 a_4 a_5 + \frac{7}{48} a_2^2 a_4 a_5 + \\
 & \frac{1}{84} \pi^2 a_3^2 a_4 a_5 + \frac{13}{72} a_2 a_3^3 a_5 + \frac{1}{168} a_4 a_5 + \frac{5}{6} a_2^2 a_3 a_5^2 + \frac{5}{36} \pi^2 a_2^2 a_3 a_5^2 + \frac{7}{288} a_3^3 a_5^2 + \frac{25 \pi^2 a_3^3 a_5^2}{3024} + \\
 & \frac{11}{36} a_2 a_3 a_4 a_5^2 + \frac{11}{756} \pi^2 a_2 a_3 a_4 a_5^2 + \frac{25 a_3 a_4^2 a_5^2}{1008} + \frac{1}{18} a_2^3 a_5 + \frac{\pi^2 a_2^2 a_5^3}{1134} + \frac{1}{189} a_3^3 a_5 + \frac{\pi^2 a_3^2 a_5^3}{3402} + \\
 & \frac{11}{756} a_2 a_4 a_5^3 + \frac{a_2^2 a_5^3}{1134} + \frac{a_3 a_5^4}{2835} + \frac{a_5^5}{149688}
 \end{aligned}$$

6th central moment of 5-term metalog (89 terms)

$$\begin{aligned}
 \mu_{6,5} := & \frac{31}{21} \pi^6 a_2^6 + \frac{77}{4} \pi^4 a_2^4 a_3^2 + \frac{155}{84} \pi^6 a_2^4 a_3^2 + \frac{25}{16} \pi^2 a_2^2 a_3^4 + \frac{91}{24} \pi^4 a_2^2 a_3^4 + \frac{31}{112} \pi^6 a_2^2 a_3^4 + \frac{a_3^6}{448} + \frac{5}{192} \pi^2 a_3^6 + \\
 & \frac{133 \pi^4 a_3^6}{2880} + \frac{31 \pi^6 a_3^6}{9408} + 7 \pi^4 a_2^5 a_4 + \frac{35}{2} \pi^2 a_2^3 a_3^2 a_4 + \frac{28}{3} \pi^4 a_2^3 a_3^2 a_4 + \frac{3}{16} a_2^4 a_3^4 a_4 + \frac{5}{3} \pi^2 a_2^4 a_3^4 a_4 + \\
 & \frac{553}{720} \pi^4 a_2 a_3^4 a_4 + 10 \pi^2 a_2^4 a_4^2 + \frac{7}{12} \pi^4 a_2^4 a_4^2 + 3 a_2^2 a_3^2 a_4^2 + \frac{65}{8} \pi^2 a_2^2 a_3^2 a_4^2 + \frac{21}{40} \pi^4 a_2^2 a_3^2 a_4^2 + \frac{5}{64} a_3^4 a_4^2 + \\
 & \frac{19}{96} \pi^2 a_3^4 a_4^2 + \frac{1}{64} \pi^4 a_3^4 a_4^2 + 5 a_2^3 a_4^3 + \frac{5}{3} \pi^2 a_2^3 a_4^3 + \frac{5}{4} a_2 a_3^2 a_4^3 + \frac{17}{24} \pi^2 a_2 a_3^2 a_4^3 + \frac{5}{4} a_2^2 a_4^4 + \frac{1}{16} \pi^2 a_2^2 a_4^4 + \\
 & \frac{19}{192} a_3^2 a_4^4 + \frac{5}{448} \pi^2 a_3^2 a_4^4 + \frac{23}{240} a_2 a_4^5 + \frac{a_4^6}{448} + 15 \pi^2 a_2^4 a_3 a_5 + \frac{35}{12} \pi^4 a_2^4 a_3 a_5 + \frac{3}{4} a_2^2 a_3^3 a_5 + \frac{35}{8} \pi^2 a_2^2 a_3^3 a_5 + \\
 & \frac{371}{360} \pi^4 a_2^2 a_3^3 a_5 + \frac{1}{64} a_3^5 a_5 + \frac{19}{288} \pi^2 a_3^5 a_5 + \frac{163 \pi^4 a_3^5 a_5}{8640} + 9 a_2^3 a_3 a_4 a_5 + \frac{65}{6} \pi^2 a_2^3 a_3 a_4 a_5 + \\
 & \frac{14}{45} \pi^4 a_2^3 a_3 a_4 a_5 + \frac{9}{8} a_2 a_3^3 a_4 a_5 + \frac{37}{24} \pi^2 a_2 a_3^3 a_4 a_5 + \frac{1}{15} \pi^4 a_2 a_3^3 a_4 a_5 + \frac{25}{4} a_2^2 a_3 a_4^2 a_5 + \frac{35}{24} \pi^2 a_2^2 a_3 a_4^2 a_5 + \\
 & \frac{19}{96} a_3^2 a_4 a_5 + \frac{163 \pi^2 a_3^2 a_4^2 a_5}{2016} + \frac{23}{24} a_2 a_3 a_4^3 a_5 + \frac{1}{21} \pi^2 a_2 a_3 a_4^3 a_5 + \frac{163 a_3 a_4^4 a_5}{4032} + 3 a_2^4 a_5^2 + \frac{5}{6} \pi^2 a_2^4 a_5^2 + \\
 & \frac{7}{180} \pi^4 a_2^4 a_5^2 + \frac{11}{8} a_2^2 a_3^2 a_5^2 + \frac{23}{24} \pi^2 a_2^2 a_3^2 a_5^2 + \frac{11}{360} \pi^4 a_2^2 a_3^2 a_5^2 + \frac{5}{192} a_3^4 a_5^2 + \frac{19}{756} \pi^2 a_3^4 a_5^2 + \frac{1}{960} \pi^4 a_3^4 a_5^2 + \\
 & \frac{5}{2} a_2^3 a_4 a_5^2 + \frac{1}{3} \pi^2 a_2^3 a_4 a_5^2 + \frac{37}{48} a_2 a_3^2 a_4 a_5^2 + \frac{43}{252} \pi^2 a_2 a_3^2 a_4 a_5^2 + \frac{17}{24} a_2^2 a_4 a_5^2 + \frac{11}{504} \pi^2 a_2^2 a_4 a_5^2 + \\
 & \frac{19}{252} a_3^2 a_4 a_5^2 + \frac{1}{224} \pi^2 a_3^2 a_4 a_5^2 + \frac{1}{14} a_2 a_4^3 a_5^2 + \frac{1}{448} a_4^4 a_5^2 + \frac{13}{48} a_2^2 a_3 a_5^3 + \frac{31}{756} \pi^2 a_2^2 a_3 a_5^3 + \frac{127 a_3^3 a_5^3}{12096} + \\
 & \frac{109 \pi^2 a_3^3 a_5^3}{45360} + \frac{25}{252} a_2 a_3 a_4 a_5^3 + \frac{2}{567} \pi^2 a_2 a_3 a_4 a_5^3 + \frac{109 a_3 a_4^2 a_5^3}{15120} + \frac{11 a_2^2 a_4^4}{1008} + \frac{\pi^2 a_2^2 a_4^4}{3024} + \frac{1}{720} a_3^2 a_5^4 + \\
 & \frac{73 \pi^2 a_3^2 a_5^4}{1197504} + \frac{47 a_2 a_4^4}{15120} + \frac{73 a_4^4 a_5^4}{399168} + \frac{139 a_3 a_5^5}{1995840} + \frac{53 a_6^5}{46702656}
 \end{aligned}$$

7th central moment of 5-term metalog (134 terms)

$$\begin{aligned}
 \mu_{7,5} := & 31 \pi^6 a_2^6 a_3 + 49 \pi^4 a_2^4 a_3^3 + \frac{1705}{72} \pi^6 a_2^4 a_3^3 + \frac{21}{16} \pi^2 a_2^2 a_3^5 + \frac{343}{48} \pi^4 a_2^2 a_3^5 + \frac{899}{360} \pi^6 a_2^2 a_3^5 + \frac{7}{384} \pi^2 a_3^7 + \\
 & \frac{49}{720} \pi^4 a_3^7 + \frac{899 \pi^6 a_3^7}{40320} + \frac{147}{2} \pi^4 a_2^5 a_3 a_4 + \frac{31}{6} \pi^6 a_2^5 a_3 a_4 + \frac{105}{4} \pi^2 a_2^3 a_3^3 a_4 + \frac{539}{12} \pi^4 a_2^3 a_3^3 a_4 + \\
 & \frac{31}{12} \pi^6 a_2^3 a_3^3 a_4 + \frac{3}{32} a_2 a_3^5 a_4 + \frac{175}{96} \pi^2 a_2 a_3^5 a_4 + \frac{3479 \pi^4 a_2 a_3^5 a_4}{1440} + \frac{31}{224} \pi^6 a_2 a_3^5 a_4 + \frac{105}{2} \pi^2 a_2^4 a_3 a_4^2 + \\
 & \frac{147}{8} \pi^4 a_2^4 a_3 a_4^2 + \frac{21}{8} a_2^3 a_3^2 a_4 + \frac{875}{48} \pi^2 a_2^2 a_3^2 a_4 + \frac{539}{90} \pi^4 a_2^2 a_3^2 a_4 + \frac{7}{128} \pi^2 a_3^5 a_4 + \frac{7}{24} \pi^2 a_3^5 a_4^2 + \\
 & \frac{203 \pi^4 a_3^5 a_4^2}{1920} + \frac{21}{2} a_2^3 a_3 a_4^3 + \frac{35}{2} \pi^2 a_2^3 a_3 a_4^3 + \frac{49}{60} \pi^4 a_2^3 a_3 a_4^3 + \frac{35}{24} a_2 a_3^3 a_4^3 + \frac{91}{36} \pi^2 a_2 a_3^3 a_4^3 + \\
 & \frac{7}{48} \pi^4 a_2 a_3^3 a_4^3 + \frac{35}{8} a_2^2 a_3 a_4^4 + \frac{35}{24} \pi^2 a_2^2 a_3 a_4^4 + \frac{7}{48} a_3 a_4^4 + \frac{29}{384} \pi^2 a_3^4 a_4 + \frac{77}{160} a_2 a_3 a_4^5 + \frac{1}{32} \pi^2 a_2 a_3 a_4^5 + \\
 & \frac{29 a_3 a_4^6}{1920} + \frac{49}{3} \pi^4 a_2^6 a_5 + 35 \pi^2 a_2^4 a_3^2 a_5 + \frac{245}{8} \pi^4 a_2^4 a_3^2 a_5 + \frac{31}{36} \pi^6 a_2^4 a_3^2 a_5 + \frac{9}{16} a_2^2 a_3^4 a_5 + \frac{105}{16} \pi^2 a_2^2 a_3^4 a_5 + \\
 & \frac{931}{180} \pi^4 a_2^2 a_3^4 a_5 + \frac{31}{168} \pi^6 a_2^2 a_3^4 a_5 + \frac{1}{128} a_3^6 a_5 + \frac{7}{96} \pi^2 a_3^6 a_5 + \frac{3157 \pi^4 a_3^6 a_5}{51840} + \frac{31 \pi^6 a_3^6 a_5}{12096} + 35 \pi^2 a_2^5 a_4 a_5 + \\
 & \frac{49}{12} \pi^4 a_2^5 a_4 a_5 + \frac{49}{4} a_2^3 a_3^2 a_4 a_5 + \frac{1085}{24} \pi^2 a_2^3 a_3^2 a_4 a_5 + \frac{931}{120} \pi^4 a_2^3 a_3^2 a_4 a_5 + \frac{63}{64} a_2 a_3^4 a_4 a_5 + \\
 & \frac{959}{288} \pi^2 a_2 a_3^4 a_4 a_5 + \frac{6209 \pi^4 a_2 a_3^4 a_4 a_5}{8640} + 21 a_2^4 a_4 a_5 + \frac{35}{3} \pi^2 a_2^4 a_4 a_5 + \frac{49}{180} \pi^4 a_2^4 a_4 a_5 + \frac{91}{8} a_2^2 a_3^2 a_4 a_5 + \\
 & \frac{497}{48} \pi^2 a_2^2 a_3^2 a_4 a_5 + \frac{7}{20} \pi^4 a_2^2 a_3^2 a_4 a_5 + \frac{7}{32} a_3^4 a_4 a_5 + \frac{451 \pi^2 a_3^4 a_4 a_5}{1728} + \frac{7}{576} \pi^4 a_3^4 a_4 a_5 + \frac{35}{4} a_2^3 a_4 a_5 +
 \end{aligned}$$

7th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{91}{72} \pi^2 a_3^3 a_4 a_5 + \frac{77}{32} a_2 a_3^2 a_4^3 a_5 + \frac{187}{288} \pi^2 a_2 a_3^2 a_4^3 a_5 + \frac{21}{16} a_2^2 a_4^4 a_5 + \frac{1}{24} \pi^2 a_2^2 a_4^4 a_5 + \frac{451 a_3^2 a_4^4 a_5}{3456} + \\
 & \frac{5}{576} \pi^2 a_3^2 a_4^4 a_5 + \frac{47}{576} a_2 a_4^5 a_5 + \frac{1}{576} a_2^6 a_4 a_5 + \frac{7}{576} a_2^4 a_3 a_4^2 a_5 + \frac{35}{3} \pi^2 a_2^4 a_3 a_4^2 a_5 + \frac{49}{40} \pi^4 a_2^4 a_3 a_4^2 a_5 + \frac{7}{4} a_2^2 a_3^2 a_5^2 + \\
 & \frac{245}{72} \pi^2 a_2^2 a_3^3 a_5 + \frac{343}{720} \pi^4 a_2^3 a_3^2 a_5 + \frac{7}{320} a_3^5 a_5^2 + \frac{11}{216} \pi^2 a_3^5 a_5^2 + \frac{2401 \pi^4 a_3^5 a_5^2}{259200} + \frac{49}{4} a_2^3 a_3 a_4 a_5^2 + \\
 & \frac{49}{9} \pi^2 a_2^3 a_3 a_4 a_5^2 + \frac{77}{540} \pi^4 a_2^3 a_3 a_4 a_5^2 + \frac{119}{96} a_2^3 a_3 a_4 a_5^2 + \frac{379}{432} \pi^2 a_2 a_3^3 a_4 a_5^2 + \frac{7}{240} \pi^4 a_2 a_3^3 a_4 a_5^2 + \\
 & \frac{413}{96} a_2^2 a_3 a_4^2 a_5^2 + \frac{97}{144} \pi^2 a_2^2 a_3 a_4^2 a_5^2 + \frac{11}{72} a_3^2 a_4^2 a_5^2 + \frac{343 \pi^2 a_3^2 a_4^2 a_5^2}{8640} + \frac{227}{432} a_2 a_3 a_4^3 a_5^2 + \frac{1}{48} \pi^2 a_2 a_3 a_4^3 a_5^2 + \\
 & \frac{343 a_3 a_4^4 a_5^2}{17280} + \frac{7}{6} a_2^4 a_5^3 + \frac{7}{18} \pi^2 a_2^4 a_5^3 + \frac{7 \pi^4 a_2^4 a_5^3}{1620} + \frac{77}{96} a_2^2 a_3^2 a_5^3 + \frac{19}{54} \pi^2 a_2^2 a_3^2 a_5^3 + \frac{7}{810} \pi^4 a_2^2 a_3^2 a_5^3 + \\
 & \frac{17 a_3^4 a_5^3}{1152} + \frac{31 \pi^2 a_3^4 a_5^3}{3240} + \frac{91 \pi^4 a_3^4 a_5^3}{285120} + \frac{49}{48} a_2^3 a_4^3 a_5^3 + \frac{11}{108} \pi^2 a_2^3 a_4^3 a_5^3 + \frac{191}{576} a_2^2 a_3 a_4^3 a_5^3 + \frac{367 \pi^2 a_2 a_3^2 a_4 a_5^3}{6480} + \\
 & \frac{1}{4} a_2^2 a_4^3 a_5^3 + \frac{1}{162} \pi^2 a_2^2 a_4^3 a_5^3 + \frac{31 a_3^2 a_4^2 a_5^3}{1080} + \frac{13 \pi^2 a_3^2 a_4^2 a_5^3}{9504} + \frac{149 a_2 a_3^3 a_4 a_5^3}{6480} + \frac{13 a_4^4 a_5^3}{19008} + \frac{25}{288} a_2^2 a_3 a_4^4 a_5^3 + \\
 & \frac{7}{720} \pi^2 a_2^2 a_3 a_4^4 a_5^3 + \frac{19 a_3^3 a_5^4}{5760} + \frac{349 \pi^2 a_3^3 a_5^4}{570240} + \frac{29 a_2 a_3 a_4 a_5^4}{1080} + \frac{73 \pi^2 a_2 a_3 a_4 a_5^4}{85536} + \frac{349 a_3 a_4^2 a_5^4}{190080} + \\
 & \frac{1}{400} a_2^5 a_5 + \frac{\pi^2 a_2^2 a_5^5}{21384} + \frac{859 a_3^2 a_5^5}{2851200} + \frac{79 \pi^2 a_3^2 a_5^5}{6671808} + \frac{179 a_2 a_4 a_5^5}{285120} + \frac{79 a_2^2 a_5^5}{2223936} + \frac{a_7^5}{6671808} +
 \end{aligned}$$

8th central moment of 5-term metalog (210 terms)

$$\begin{aligned}
 \mu_{8,5} := & \frac{127}{15} \pi^8 a_2^8 + \frac{2263}{9} \pi^6 a_2^6 a_3^2 + \frac{889}{45} \pi^8 a_2^6 a_3^2 + \frac{735}{8} \pi^4 a_2^4 a_3^4 + \frac{4805}{36} \pi^6 a_2^4 a_3^4 + \frac{889}{120} \pi^8 a_2^4 a_3^4 + \\
 & \frac{49}{48} \pi^2 a_2^2 a_3^6 + \frac{1519}{144} \pi^4 a_2^2 a_3^6 + \frac{22847}{2160} \pi^6 a_2^2 a_3^6 + \frac{127}{240} \pi^8 a_2^2 a_3^6 + \frac{a_3^8}{2304} + \frac{7}{576} \pi^2 a_3^8 + \frac{1421}{17280} \pi^4 a_3^8 + \\
 & \frac{40}{544320} 331 \pi^6 a_3^8 + \frac{127}{34560} \pi^8 a_3^8 + \frac{124}{3} \pi^6 a_2^7 a_4 + 343 \pi^4 a_2^5 a_3^2 a_4 + 124 \pi^6 a_2^5 a_3^2 a_4 + \frac{385}{12} \pi^2 a_2^3 a_3^4 a_4 + \\
 & \frac{392}{3} \pi^4 a_2^3 a_3^4 a_4 + \frac{3875}{108} \pi^6 a_2^3 a_3^4 a_4 + \frac{1}{16} a_2 a_3^6 a_4 + \frac{7}{4} \pi^2 a_2 a_3^6 a_4 + \frac{245}{48} \pi^4 a_2 a_3^6 a_4 + \frac{3379}{2520} \pi^6 a_2 a_3^6 a_4 + \\
 & \frac{196}{3} \pi^4 a_2^6 a_4^2 + \frac{31}{9} \pi^6 a_2^6 a_4^2 + 140 \pi^2 a_2^4 a_3^2 a_4^2 + \frac{2009}{12} \pi^4 a_2^4 a_3^2 a_4^2 + \frac{31}{4} \pi^6 a_2^4 a_3^2 a_4^2 + \frac{9}{4} a_2^2 a_3^4 a_4^2 + \\
 & \frac{1435}{48} \pi^2 a_2^2 a_3^4 a_4^2 + \frac{10633}{360} \pi^4 a_2^2 a_3^4 a_4^2 + \frac{155}{112} \pi^6 a_2^2 a_3^4 a_4^2 + \frac{7}{192} a_3^6 a_4^2 + \frac{203}{576} \pi^2 a_3^6 a_4^2 + \frac{9107}{25920} \pi^4 a_3^6 a_4^2 + \\
 & \frac{31}{1728} \pi^6 a_3^6 a_4^2 + \frac{140}{3} \pi^2 a_2^5 a_4^3 + \frac{98}{9} \pi^4 a_2^5 a_4^3 + \frac{49}{3} a_2^3 a_3^2 a_4^3 + \frac{665}{9} \pi^2 a_2^3 a_3^2 a_4^3 + \frac{4753}{270} \pi^4 a_2^3 a_3^2 a_4^3 + \\
 & \frac{35}{24} a_2 a_3^4 a_4^3 + \frac{413}{72} \pi^2 a_2 a_3^4 a_4^3 + \frac{14}{9} \pi^4 a_2 a_3^4 a_4^3 + 14 a_2^4 a_4^3 + \frac{35}{3} \pi^2 a_2^4 a_4^3 + \frac{49}{120} \pi^4 a_2^4 a_4^3 + \frac{35}{4} a_2^2 a_3^2 a_4^3 + \\
 & \frac{161}{16} \pi^2 a_2^2 a_3^4 a_4 + \frac{7}{16} \pi^4 a_2^2 a_3^4 a_4 + \frac{203}{1152} a_3^4 a_4^4 + \frac{1301}{5184} \pi^2 a_3^4 a_4^4 + \frac{49}{3456} \pi^4 a_3^4 a_4^4 + \frac{14}{3} a_2^3 a_5^5 + \frac{161}{180} \pi^2 a_2^3 a_4^5 + \\
 & \frac{301}{240} a_2 a_3^5 a_4 + \frac{17}{40} \pi^2 a_2 a_3^5 a_4 + \frac{49}{90} a_2^2 a_4^6 + \frac{1}{48} \pi^2 a_2^2 a_4^6 + \frac{1301}{25920} a_3^6 a_4^6 + \frac{7}{420} \pi^2 a_3^6 a_4^6 + \frac{11}{420} a_2 a_4^7 + \frac{a_4^8}{2304} + \\
 & \frac{490}{3} \pi^4 a_2^6 a_3 a_5 + \frac{217}{9} \pi^6 a_2^6 a_3 a_5 + \frac{175}{3} \pi^2 a_2^4 a_3^3 a_5 + \frac{1715}{12} \pi^4 a_2^4 a_3^3 a_5 + \frac{2449}{108} \pi^6 a_2^4 a_3^3 a_5 + \frac{3}{8} a_2^2 a_3^5 a_5 + \\
 & \frac{385}{48} \pi^2 a_2^2 a_3^5 a_5 + \frac{5537}{360} \pi^4 a_2^2 a_3^5 a_5 + \frac{7967}{3024} \pi^6 a_2^2 a_3^5 a_5 + \frac{1}{192} a_3^7 a_5 + \frac{203}{2880} \pi^2 a_3^7 a_5 + \frac{679}{5184} \pi^4 a_3^7 a_5 + \\
 & \frac{279}{11200} \pi^6 a_3^7 a_5 + 196 \pi^2 a_2^5 a_3 a_4 a_5 + \frac{343}{3} \pi^4 a_2^5 a_3 a_4 a_5 + \frac{124}{45} \pi^6 a_2^5 a_3 a_4 a_5 + 13 a_2^3 a_3^3 a_4 a_5 +
 \end{aligned}$$

8th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{665}{6} \pi^2 a_2^3 a_3^3 a_4 a_5 + \frac{5831}{90} \pi^4 a_2^3 a_3^3 a_4 a_5 + \frac{124}{63} \pi^6 a_2^3 a_3^3 a_4 a_5 + \frac{13}{16} a_2 a_3^5 a_4 a_5 + \frac{385}{72} \pi^2 a_2 a_3^5 a_4 a_5 + \\
 & \frac{497}{144} \pi^4 a_2 a_3^5 a_4 a_5 + \frac{31}{252} \pi^6 a_2 a_3^5 a_4 a_5 + 56 a_2^4 a_3 a_4^2 a_5 + \frac{385}{3} \pi^2 a_2^4 a_3 a_4^2 a_5 + \frac{2989}{180} \pi^4 a_2^4 a_3 a_4^2 a_5 + \\
 & \frac{63}{4} a_2^3 a_3^2 a_4 a_5 + \frac{2695}{72} \pi^2 a_2^2 a_3^2 a_4^2 a_5 + \frac{6713}{1080} \pi^4 a_2^2 a_3^2 a_4^2 a_5 + \frac{203}{960} a_3^5 a_4 a_5 + \frac{485}{864} \pi^2 a_3^5 a_4 a_5 + \\
 & \frac{189 \pi^4 a_3^5 a_4 a_5}{1600} + \frac{119}{3} a_2^3 a_3^3 a_4 a_5 + \frac{413}{18} \pi^2 a_2^3 a_3^3 a_4 a_5 + \frac{28}{45} \pi^4 a_2^3 a_3^3 a_4 a_5 + \frac{301}{72} a_2 a_3^3 a_4 a_5 + \\
 & \frac{2293}{648} \pi^2 a_2 a_3^3 a_4 a_5 + \frac{7}{54} \pi^4 a_2 a_3^3 a_4 a_5 + \frac{49}{6} a_2^2 a_3^4 a_4 a_5 + \frac{211}{144} \pi^2 a_2^2 a_3^4 a_4 a_5 + \frac{485}{1728} a_3^4 a_4 a_5 + \\
 & \frac{27}{320} \pi^2 a_3^4 a_4 a_5 + \frac{1391 a_2 a_3 a_4^5 a_5}{2160} + \frac{1}{36} \pi^2 a_2 a_3 a_4^5 a_5 + \frac{27 a_3 a_4^6 a_5}{1600} + 28 \pi^2 a_2^6 a_5 + \frac{49}{9} \pi^4 a_2^6 a_5 + \\
 & \frac{31}{135} \pi^6 a_2^6 a_5 + 11 a_2^4 a_3^2 a_5 + \frac{105}{2} \pi^2 a_2^4 a_3^2 a_5 + \frac{539}{30} \pi^4 a_2^4 a_3^2 a_5 + \frac{341}{16} \pi^6 a_2^4 a_3^2 a_5 + \frac{29}{16} a_2^2 a_3^4 a_5 + \\
 & \frac{1127}{144} \pi^2 a_2^2 a_3^4 a_5 + \frac{7147 \pi^4 a_2^2 a_3^4 a_5}{2160} + \frac{31}{336} \pi^6 a_2^2 a_3^4 a_5 + \frac{13}{720} a_3^6 a_5 + \frac{137 \pi^2 a_3^6 a_5}{1728} + \frac{1967 \pi^4 a_3^6 a_5}{48600} + \\
 & \frac{527 \pi^6 a_3^6 a_5}{399168} + 28 a_2^5 a_4 a_5 + \frac{70}{3} \pi^2 a_2^5 a_4 a_5 + \frac{98}{45} \pi^4 a_2^5 a_4 a_5 + \frac{161}{6} a_2^3 a_3^2 a_4 a_5 + \frac{217}{6} \pi^2 a_2^3 a_3^2 a_4 a_5 + \\
 & \frac{553}{135} \pi^4 a_2^3 a_3^2 a_4 a_5 + \frac{77}{48} a_2 a_3^4 a_4 a_5 + \frac{74}{27} \pi^2 a_2 a_3^4 a_4 a_5 + \frac{259}{648} \pi^4 a_2 a_3^4 a_4 a_5 + 21 a_2^4 a_4 a_5 + \frac{119}{18} \pi^2 a_2^4 a_4 a_5 + \\
 & \frac{77}{540} \pi^4 a_2^4 a_4 a_5 + \frac{77}{6} a_2^2 a_3^2 a_4 a_5 + \frac{155}{24} \pi^2 a_2^2 a_3^2 a_4 a_5 + \frac{7}{40} \pi^4 a_2^2 a_3^2 a_4 a_5 + \frac{137}{576} a_3^4 a_4 a_5 + \frac{281 \pi^2 a_3^4 a_4 a_5}{1620} + \\
 & \frac{119 \pi^4 a_3^4 a_4 a_5}{19008} + \frac{217}{36} a_2^3 a_4 a_5 + \frac{2}{3} \pi^2 a_2^3 a_4 a_5 + \frac{52}{27} a_2 a_3^3 a_4 a_5 + \frac{97}{270} \pi^2 a_2 a_3^3 a_4 a_5 + \frac{335}{432} a_2^4 a_4 a_5 +
 \end{aligned}$$

8th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{1}{48} \pi^2 a_2^2 a_4^2 a_5^2 + \frac{281 a_3^2 a_4^2 a_5^2}{3240} + \frac{85 \pi^2 a_3^2 a_4^2 a_5^2}{19008} + \frac{239 a_2 a_4^2 a_5^2}{5400} + \frac{17 a_4^6 a_5^2}{19008} + \frac{7 a_2^4 a_3 a_5^3}{19008} + \frac{21 \pi^2 a_2^4 a_3 a_5^3}{4} + \\
 & \frac{371}{810} \pi^4 a_2^4 a_3 a_5^3 + \frac{217 a_2^2 a_3^3 a_5^3}{144} + \frac{1073 \pi^2 a_2^2 a_3^3 a_5^3}{648} + \frac{175 a_2^2 a_3^3 a_5^3}{972} + \frac{467 a_3^5 a_5^3}{25920} + \frac{2003 \pi^2 a_3^5 a_5^3}{77760} + \\
 & \frac{23051 \pi^4 a_3^5 a_5^3}{6415200} + \frac{245 a_2^3 a_3 a_4 a_5^3}{36} + \frac{61 \pi^2 a_2^3 a_3 a_4 a_5^3}{27} + \frac{56 \pi^4 a_2^3 a_3 a_4 a_5^3}{1215} + \frac{331 a_2 a_3^3 a_4 a_5^3}{432} + \\
 & \frac{121}{324} \pi^2 a_2 a_3^3 a_4 a_5^3 + \frac{91 \pi^4 a_2 a_3^3 a_4 a_5^3}{8910} + \frac{97 a_2^2 a_3 a_4 a_5^3}{48} + \frac{407 \pi^2 a_2^2 a_3 a_4 a_5^3}{1620} + \frac{2003 a_3^3 a_4 a_5^3}{25920} + \\
 & \frac{3293 \pi^2 a_3^3 a_4 a_5^3}{213840} + \frac{119 a_2 a_3 a_4 a_5^3}{540} + \frac{13 \pi^2 a_2 a_3 a_4 a_5^3}{1782} + \frac{3293 a_3 a_4 a_5^3}{427680} + \frac{35 a_4^4 a_5^3}{72} + \frac{11 \pi^2 a_2^4 a_5^3}{108} + \\
 & \frac{7 \pi^4 a_2^4 a_5^3}{3240} + \frac{23 a_2^2 a_3 a_5^3}{72} + \frac{79 \pi^2 a_2^2 a_3 a_5^3}{720} + \frac{511 \pi^4 a_2^2 a_3 a_5^3}{213840} + \frac{41 a_3^4 a_5^3}{6480} + \frac{1321 \pi^2 a_3^4 a_5^3}{427680} + \frac{1967 \pi^4 a_3^4 a_5^3}{22239360} + \\
 & \frac{1}{3} \frac{3 a_2^4 a_5^3}{1620} + \frac{47 \pi^2 a_2^3 a_4 a_5^3}{1620} + \frac{17 a_2 a_3^2 a_4 a_5^3}{144} + \frac{1753 \pi^2 a_2 a_3^2 a_4 a_5^3}{106920} + \frac{163 a_2^2 a_4^2 a_5^3}{2160} + \frac{73 \pi^2 a_2^2 a_4^2 a_5^3}{42768} + \\
 & \frac{1321 a_3^2 a_4^2 a_5^3}{142560} + \frac{281 \pi^2 a_3^2 a_4^2 a_5^3}{741312} + \frac{353 a_2 a_3 a_4 a_5^3}{53460} + \frac{281 a_4^4 a_5^3}{1482624} + \frac{41 a_2^2 a_3 a_5^3}{1800} + \frac{497 \pi^2 a_2^2 a_3 a_5^3}{213840} + \frac{4039 a_3^3 a_5^3}{4276800} + \\
 & \frac{34213 \pi^2 a_3^3 a_5^3}{233513280} + \frac{2413 a_2 a_3 a_4 a_5^3}{356400} + \frac{79 \pi^2 a_2 a_3 a_4 a_5^3}{416988} + \frac{34213 a_3 a_4 a_5^3}{77837760} + \frac{53 a_2^2 a_5^3}{106920} + \frac{53 \pi^2 a_2^2 a_5^3}{5003856} + \\
 & \frac{1148003 a_3^2 a_5^3}{17513496000} + \frac{137 \pi^2 a_2^2 a_3 a_5^3}{60046272} + \frac{3623 a_2 a_4 a_5^3}{29189160} + \frac{137 a_4^2 a_5^3}{20015424} + \frac{361 a_3 a_5^3}{175134960} + \frac{95 a_8^3}{4083146496} + \\
 & \frac{17513496000}{17513496000} + \frac{60046272}{60046272} + \frac{29189160}{29189160} + \frac{20015424}{20015424} + \frac{175134960}{175134960} + \frac{4083146496}{4083146496}
 \end{aligned}$$

9th central moment of 5-term metalog (297 terms)

$$\begin{aligned}
 \mu_{9,5} := & \frac{1524}{5} \pi^8 a_2^8 a_3 + 1178 \pi^6 a_2^6 a_3^3 + \frac{889}{2} \pi^8 a_2^6 a_3^3 + \frac{2793}{20} \pi^4 a_2^4 a_3^5 + \frac{3751}{8} \pi^6 a_2^4 a_3^5 + \frac{24003}{200} \pi^8 a_2^4 a_3^5 + \\
 & \frac{3}{4} \pi^2 a_2^2 a_3^7 + \frac{2107}{160} \pi^4 a_2^2 a_3^7 + \frac{7037}{240} \pi^6 a_2^2 a_3^7 + \frac{36957}{5600} \pi^8 a_2^2 a_3^7 + \frac{1}{128} \pi^2 a_3^9 + \frac{833 \pi^4 a_3^9}{9600} + \frac{20243 \pi^6 a_3^9}{120960} + \\
 & \frac{12319 \pi^8 a_3^9}{336000} + 806 \pi^6 a_2^7 a_3 a_4 + \frac{254}{5} \pi^8 a_2^7 a_3 a_4 + \frac{10143}{10} \pi^4 a_2^5 a_3^3 a_4 + \frac{2263}{2} \pi^6 a_2^5 a_3^3 a_4 + \\
 & \frac{2667}{50} \pi^8 a_2^5 a_3^3 a_4 + \frac{273}{8} \pi^2 a_2^3 a_3^5 a_4 + \frac{2205}{8} \pi^4 a_2^3 a_3^5 a_4 + \frac{26443}{120} \pi^6 a_2^3 a_3^5 a_4 + \frac{381}{40} \pi^8 a_2^3 a_3^5 a_4 + \\
 & \frac{1}{32} a_2 a_3^7 a_4 + \frac{49}{32} \pi^2 a_2 a_3^7 a_4 + \frac{2009}{240} \pi^4 a_2 a_3^7 a_4 + \frac{185597 \pi^6 a_2 a_3^7 a_4}{30240} + \frac{127}{480} \pi^8 a_2 a_3^7 a_4 + \\
 & 735 \pi^4 a_2^6 a_3 a_4^2 + \frac{403}{2} \pi^6 a_2^6 a_3 a_4^2 + \frac{525}{2} \pi^2 a_2^4 a_3^3 a_4^2 + \frac{6321}{8} \pi^4 a_2^4 a_3^3 a_4^2 + \frac{2077}{12} \pi^6 a_2^4 a_3^3 a_4^2 + \\
 & \frac{27}{16} a_2 a_3^5 a_4 + \frac{1281}{32} \pi^2 a_2 a_3^5 a_4 + \frac{1813}{20} \pi^4 a_2 a_3^5 a_4 + \frac{21669 \pi^6 a_2 a_3^5 a_4}{1120} + \frac{3}{128} a_3^7 a_4 + \frac{119}{320} \pi^2 a_3^7 a_4 + \\
 & \frac{4571 \pi^4 a_3^7 a_4^2}{5760} + \frac{3007 \pi^6 a_3^7 a_4^2}{16800} + 294 \pi^2 a_2^5 a_3 a_4^3 + 245 \pi^4 a_2^5 a_3 a_4^3 + \frac{93}{10} \pi^6 a_2^5 a_3 a_4^3 + \frac{39}{2} a_2^3 a_3^3 a_4 + \\
 & \frac{385}{2} \pi^2 a_2^3 a_3^3 a_4 + \frac{1421}{10} \pi^4 a_2^3 a_3^3 a_4 + \frac{155}{28} \pi^6 a_2^3 a_3^3 a_4 + \frac{21}{16} a_2 a_3^5 a_4 + \frac{315}{32} \pi^2 a_2 a_3^5 a_4 + \frac{5467}{720} \pi^4 a_2 a_3^5 a_4 + \\
 & \frac{31}{96} \pi^6 a_2 a_3^5 a_4 + 42 a_2^4 a_3 a_4^4 + \frac{245}{2} \pi^2 a_2^4 a_3 a_4^4 + \frac{2597}{120} \pi^4 a_2^4 a_3 a_4^4 + \frac{105}{8} a_2^2 a_3^4 a_4 + \frac{595}{16} \pi^2 a_2^2 a_3^4 a_4 + \\
 & \frac{245}{32} \pi^4 a_2^2 a_3^4 a_4 + \frac{119}{640} a_3^5 a_4 + \frac{653 \pi^2 a_3^5 a_4}{1152} + \frac{679 \pi^4 a_3^5 a_4}{4800} + 21 a_2^3 a_3 a_4^5 + \frac{623}{40} \pi^2 a_2^3 a_3 a_4^5 + \\
 & \frac{21}{40} \pi^4 a_2^3 a_3 a_4^5 + \frac{371}{160} a_2 a_3^3 a_4^5 + \frac{677}{288} \pi^2 a_2 a_3^3 a_4^5 + \frac{49}{480} \pi^4 a_2 a_3^3 a_4^5 + \frac{133}{40} a_2^2 a_3 a_4^6 + \frac{117}{160} \pi^2 a_2^2 a_3 a_4^6 + \\
 & \frac{653 a_3^3 a_4^6}{5760} + \frac{97 \pi^2 a_3^3 a_4^6}{2400} + \frac{521 a_2 a_3 a_4^7}{2520} + \frac{1}{96} \pi^2 a_2 a_3 a_4^7 + \frac{97 a_3 a_4^8}{22400} + 124 \pi^6 a_2^8 a_5 + \frac{3822}{5} \pi^4 a_2^6 a_3 a_5 +
 \end{aligned}$$

9th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{2821}{6} \pi^6 a_2^6 a_3^2 a_5 + \frac{889}{75} \pi^8 a_2^6 a_3^2 a_5 + \frac{315}{4} \pi^2 a_2^4 a_3^4 a_5 + \frac{3381}{8} \pi^4 a_2^4 a_3^4 a_5 + \frac{5177}{24} \pi^6 a_2^4 a_3^4 a_5 + \\
 & \frac{127}{20} \pi^8 a_2^4 a_3^4 a_5 + \frac{1}{4} a_2^2 a_3^6 a_5 + \frac{273}{32} \pi^2 a_2^2 a_3^6 a_5 + \frac{7889}{240} \pi^4 a_2^2 a_3^6 a_5 + \frac{500\,929 \pi^6 a_2^2 a_3^6 a_5}{30\,240} + \\
 & \frac{127}{240} \pi^8 a_2^2 a_3^6 a_5 + \frac{1}{384} a_3^8 a_5 + \frac{119 \pi^2 a_3^8 a_5}{1920} + \frac{1253 \pi^4 a_3^8 a_5}{5760} + \frac{209\,281 \pi^6 a_3^8 a_5}{1814\,400} + \frac{127 \pi^8 a_3^8 a_5}{31\,680} + \\
 & 294 \pi^4 a_2^7 a_4 a_5 + 31 \pi^6 a_2^7 a_4 a_5 + 567 \pi^2 a_2^5 a_3^2 a_4 a_5 + \frac{3969}{4} \pi^4 a_2^5 a_3^2 a_4 a_5 + \frac{2573}{20} \pi^6 a_2^5 a_3^2 a_4 a_5 + \\
 & \frac{99}{8} a_2^3 a_3^4 a_4 a_5 + \frac{3185}{16} \pi^2 a_2^3 a_3^4 a_4 a_5 + \frac{11\,711}{40} \pi^4 a_2^3 a_3^4 a_4 a_5 + \frac{42\,067 \pi^6 a_2^3 a_3^4 a_4 a_5}{1008} + \frac{39}{64} a_2 a_3^6 a_4 a_5 + \\
 & \frac{2261}{320} \pi^2 a_2 a_3^6 a_4 a_5 + \frac{30\,079 \pi^4 a_2 a_3^6 a_4 a_5}{2880} + \frac{55\,831 \pi^6 a_2 a_3^6 a_4 a_5}{33\,600} + 252 \pi^2 a_2^6 a_4^2 a_5 + 98 \pi^4 a_2^6 a_4^2 a_5 + \\
 & \frac{31}{15} \pi^6 a_2^6 a_4^2 a_5 + 99 a_2^4 a_3^2 a_4^2 a_5 + \frac{1155}{2} \pi^2 a_2^4 a_3^2 a_4^2 a_5 + \frac{10\,143}{40} \pi^4 a_2^4 a_3^2 a_4^2 a_5 + \frac{93}{14} \pi^6 a_2^4 a_3^2 a_4^2 a_5 + \\
 & 18 a_2^4 a_3^2 a_4^2 a_5 + \frac{721}{8} \pi^2 a_2^2 a_3^4 a_4^2 a_5 + \frac{10\,871}{240} \pi^4 a_2^2 a_3^4 a_4^2 a_5 + \frac{155}{112} \pi^6 a_2^2 a_3^4 a_4^2 a_5 + \frac{119}{640} a_3^6 a_4^2 a_5 + \\
 & \frac{179}{192} \pi^2 a_3^6 a_4^2 a_5 + \frac{47\,257 \pi^4 a_3^6 a_4^2 a_5}{86\,400} + \frac{31 \pi^6 a_3^6 a_4^2 a_5}{1584} + 84 a_2^5 a_4^3 a_5 + 105 \pi^2 a_2^5 a_4^3 a_5 + \frac{637}{60} \pi^4 a_2^5 a_4^3 a_5 + \\
 & \frac{371}{4} a_2^3 a_3^2 a_4^3 a_5 + \frac{3689}{24} \pi^2 a_2^3 a_3^2 a_4^3 a_5 + \frac{7217}{360} \pi^4 a_2^3 a_3^2 a_4^3 a_5 + \frac{371}{64} a_2 a_3^4 a_4^3 a_5 + \frac{1109}{96} \pi^2 a_2 a_3^4 a_4^3 a_5 + \\
 & \frac{1841}{960} \pi^4 a_2 a_3^4 a_4^3 a_5 + 42 a_2^4 a_4^4 a_5 + \frac{63}{4} \pi^2 a_2^4 a_4^4 a_5 + \frac{7}{20} \pi^4 a_2^4 a_4^4 a_5 + \frac{399}{16} a_2^2 a_3^2 a_4^4 a_5 + \frac{1427}{96} \pi^2 a_2^2 a_3^2 a_4^4 a_5 +
 \end{aligned}$$

9th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{7}{16} \pi^4 a_2^2 a_3^2 a_4^2 a_5 + \frac{179}{384} a_3^4 a_4 a_5 + \frac{6751 \pi^2 a_3^4 a_4^2 a_5}{17280} + \frac{49 \pi^4 a_3^4 a_4^2 a_5}{3168} + \frac{161}{20} a_2^3 a_4^5 a_5 + \frac{47}{48} \pi^2 a_2^3 a_4^5 a_5 + \\
 & \frac{2357}{960} a_2 a_3^2 a_4^5 a_5 + \frac{829 \pi^2 a_2 a_3^2 a_4^5 a_5}{1600} + \frac{131}{180} a_2^2 a_4 a_5 + \frac{1}{48} \pi^2 a_2^2 a_4 a_5 + \frac{6751 a_2^3 a_4^6 a_5}{86400} + \frac{7 \pi^2 a_2^3 a_4^6 a_5}{1584} + \\
 & \frac{49 a_2 a_4^7 a_5}{1600} + \frac{a_4^8 a_5}{2112} + 168 \pi^2 a_2^6 a_3 a_5^2 + 147 \pi^4 a_2^6 a_3 a_5^2 + \frac{403}{30} \pi^6 a_2^6 a_3 a_5^2 + \frac{27}{2} a_2^4 a_3^3 a_5^2 + 140 \pi^2 a_2^4 a_3^3 a_5^2 + \\
 & \frac{10339}{80} \pi^4 a_2^4 a_3^3 a_5^2 + \frac{2263}{168} \pi^6 a_2^4 a_3^3 a_5^2 + \frac{27}{16} a_2^2 a_3^5 a_5^2 + \frac{2177}{160} \pi^2 a_2^2 a_3^5 a_5^2 + \frac{441}{32} \pi^4 a_2^2 a_3^5 a_5^2 + \\
 & \frac{82801 \pi^6 a_2^6 a_3^5 a_5^2}{50400} + \frac{17 a_3^7 a_5^2}{1280} + \frac{593 \pi^2 a_3^7 a_5^2}{5760} + \frac{6727 \pi^4 a_3^7 a_5^2}{2217600} + \frac{35681 \pi^6 a_3^7 a_5^2}{2217600} + 90 a_2^5 a_3 a_4 a_5^2 + \\
 & 273 \pi^2 a_2^5 a_3 a_4 a_5^2 + \frac{147}{2} \pi^4 a_2^5 a_3 a_4 a_5^2 + \frac{341}{210} \pi^6 a_2^5 a_3 a_4 a_5^2 + \frac{171}{4} a_2^3 a_3^3 a_4 a_5^2 + \frac{539}{4} \pi^2 a_2^3 a_3^3 a_4 a_5^2 + \\
 & \frac{2044}{45} \pi^4 a_2^3 a_3^3 a_4 a_5^2 + \frac{31}{28} \pi^6 a_2^3 a_3^3 a_4 a_5^2 + \frac{141}{80} a_2 a_3^5 a_4 a_5^2 + \frac{587}{96} \pi^2 a_2 a_3^5 a_4 a_5^2 + \frac{18137 \pi^4 a_2 a_3^5 a_4 a_5^2}{7200} + \\
 & \frac{527 \pi^6 a_2 a_3^5 a_4 a_5^2}{7392} + \frac{231}{2} a_2^4 a_3 a_4^2 a_5^2 + \frac{847}{8} \pi^2 a_2^4 a_3 a_4^2 a_5^2 + \frac{1183}{120} \pi^4 a_2^4 a_3 a_4^2 a_5^2 + \frac{819}{32} a_2^2 a_3^2 a_4^2 a_5^2 + \\
 & \frac{521}{16} \pi^2 a_2^2 a_3^2 a_4^2 a_5^2 + \frac{2779}{720} \pi^4 a_2^2 a_3^2 a_4^2 a_5^2 + \frac{593 a_3^5 a_4^2 a_5^2}{1920} + \frac{961 \pi^2 a_3^5 a_4^2 a_5^2}{1920} + \frac{8057 \pi^4 a_3^5 a_4^2 a_5^2}{105600} + \\
 & \frac{343}{8} a_2^3 a_3^3 a_4^2 a_5^2 + \frac{281}{18} \pi^2 a_2^3 a_3^3 a_4^2 a_5^2 + \frac{7}{20} \pi^4 a_2^3 a_3^3 a_4^2 a_5^2 + \frac{75}{16} a_2 a_3^3 a_4^3 a_5^2 + \frac{1105}{432} \pi^2 a_2 a_3^3 a_4^3 a_5^2 + \\
 & \frac{119 \pi^4 a_2 a_3^3 a_4^3 a_5^2}{1584} + \frac{643}{96} a_2^2 a_3 a_4^4 a_5^2 + \frac{433}{480} \pi^2 a_2^2 a_3 a_4^4 a_5^2 + \frac{961 a_3^3 a_4^4 a_5^2}{3840} + \frac{1151 \pi^2 a_3^3 a_4^4 a_5^2}{21120} + \\
 & \frac{3277 a_2 a_3 a_4^5 a_5^2}{7200} + \frac{17 \pi^2 a_2 a_3 a_4^5 a_5^2}{1056} + \frac{1151 a_3 a_4^6 a_5^2}{105600} + 12 a_2^6 a_5^3 + 14 \pi^2 a_2^6 a_5^3 + \frac{49}{15} \pi^4 a_2^6 a_5^3 + \\
 & \frac{31}{945} \pi^6 a_2^6 a_5^3 + 18 a_2^4 a_3^2 a_5^3 + \frac{273}{8} \pi^2 a_2^4 a_3^2 a_5^3 + \frac{42}{5} \pi^4 a_2^4 a_3^2 a_5^3 + \frac{31}{189} \pi^6 a_2^4 a_3^2 a_5^3 + \frac{71}{32} a_2^2 a_3^4 a_5^3 +
 \end{aligned}$$

9th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{187}{36} \pi^2 a_2^2 a_3^4 a_5^3 + \frac{679}{432} \pi^4 a_2^2 a_3^4 a_5^3 + \frac{403}{11088} \pi^6 a_2^2 a_3^4 a_5^3 + \frac{109}{5760} \pi^6 a_3^6 a_5^3 + \frac{911}{17280} \pi^2 a_3^6 a_5^3 + \frac{14021}{712800} \pi^4 a_3^6 a_5^3 + \\
 & \frac{155}{288288} \pi^6 a_3^6 a_5^3 + 21 a_2^5 a_4 a_5^3 + \frac{49}{4} \pi^2 a_2^5 a_4 a_5^3 + \frac{77}{90} \pi^4 a_2^5 a_4 a_5^3 + \frac{371}{16} a_2^3 a_3^2 a_4 a_5^3 + \frac{153}{8} \pi^2 a_2^3 a_3^2 a_4 a_5^3 + \\
 & \frac{931}{540} \pi^4 a_2^3 a_3^2 a_4 a_5^3 + \frac{773}{576} a_2 a_3^4 a_4 a_5^3 + \frac{12991}{8640} \pi^2 a_2 a_3^4 a_4 a_5^3 + \frac{24689}{142560} \pi^4 a_2 a_3^4 a_4 a_5^3 + \frac{49}{4} a_2^4 a_4 a_5^3 + \\
 & 3 \pi^2 a_2^4 a_4 a_5^3 + \frac{7}{135} \pi^4 a_2^4 a_4 a_5^3 + \frac{261}{32} a_2^2 a_3^2 a_4 a_5^3 + \frac{1091}{360} \pi^2 a_2^2 a_3^2 a_4 a_5^3 + \frac{91}{1320} \pi^4 a_2^2 a_3^2 a_4 a_5^3 + \\
 & \frac{911}{5760} a_2^4 a_3^2 a_4 a_5^3 + \frac{2003}{23760} \pi^2 a_2^4 a_3^2 a_4 a_5^3 + \frac{35}{13728} \pi^4 a_2^4 a_3^2 a_4 a_5^3 + \frac{145}{48} a_2^3 a_3^3 a_4 a_5^3 + \frac{149}{540} \pi^2 a_2^3 a_3^3 a_4 a_5^3 + \frac{599}{576} a_2^2 a_3^3 a_4 a_5^3 + \\
 & \frac{3683}{23760} \pi^2 a_2^2 a_3^3 a_4 a_5^3 + \frac{253}{720} a_2 a_4 a_5^3 + \frac{13}{1584} \pi^2 a_2 a_4 a_5^3 + \frac{2003}{47520} a_2^2 a_3^2 a_4 a_5^3 + \frac{25}{13728} \pi^2 a_2^2 a_3^2 a_4 a_5^3 + \frac{4463}{237600} a_2 a_4 a_5^3 + \\
 & \frac{5}{13728} a_4^3 a_5^3 + \frac{7}{2} a_2^4 a_3 a_5^4 + \frac{49}{24} \pi^2 a_2^4 a_3 a_5^4 + \frac{77}{540} \pi^4 a_2^4 a_3 a_5^4 + \frac{59}{72} a_2^2 a_3^3 a_5^4 + \frac{469}{720} \pi^2 a_2^2 a_3^3 a_5^4 + \\
 & \frac{8351}{142560} \pi^4 a_2^2 a_3^3 a_5^4 + \frac{17}{1728} a_3^5 a_5^4 + \frac{37}{3564} \pi^2 a_3^5 a_5^4 + \frac{133171}{111196800} \pi^4 a_3^5 a_5^4 + \frac{71}{24} a_2^3 a_3 a_4 a_5^4 + \frac{31}{40} \pi^2 a_2^3 a_3 a_4 a_5^4 + \\
 & \frac{511}{35640} \pi^4 a_2^3 a_3 a_4 a_5^4 + \frac{499}{1440} a_2 a_3^3 a_4 a_5^4 + \frac{6331}{47520} \pi^2 a_2 a_3^3 a_4 a_5^4 + \frac{1967}{617760} \pi^4 a_2 a_3^3 a_4 a_5^4 + \frac{371}{480} a_2^2 a_3 a_4 a_5^4 + \\
 & \frac{3871}{47520} \pi^2 a_2^2 a_3 a_4 a_5^4 + \frac{37}{1188} a_3^3 a_4 a_5^4 + \frac{133171}{25945920} \pi^2 a_3^3 a_4 a_5^4 + \frac{3689}{47520} a_2 a_3 a_4 a_5^4 + \frac{281}{123552} \pi^2 a_2 a_3 a_4 a_5^4 +
 \end{aligned}$$

9th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{133171 a_3 a_4 a_5^4}{51891840} + \frac{3 a_2^4 a_5^5}{20} + \frac{3 \pi^2 a_2^4 a_5^5}{100} + \frac{7 \pi^4 a_2^4 a_5^5}{17820} + \frac{131 a_2^2 a_3^2 a_5^5}{1200} + \frac{7339 \pi^2 a_2^2 a_3^2 a_5^5}{237600} + \\
 & \frac{553 \pi^4 a_2^2 a_3^2 a_5^5}{926640} + \frac{2107 a_3^4 a_5^5}{950400} + \frac{3468541 \pi^2 a_3^4 a_5^5}{3891888000} + \frac{7 \pi^4 a_3^4 a_5^5}{308880} + \frac{59 a_2^3 a_4 a_5^5}{600} + \frac{179 \pi^2 a_2^3 a_4 a_5^5}{23760} + \\
 & \frac{1919 a_2 a_3^2 a_4 a_5^5}{52800} + \frac{113699 \pi^2 a_2 a_3^2 a_4 a_5^5}{25945920} + \frac{827 a_2^2 a_4^2 a_5^5}{39600} + \frac{79 \pi^2 a_2^2 a_4^2 a_5^5}{185328} + \frac{3468541 a_2^2 a_3^2 a_4^2 a_5^5}{1297296000} + \\
 & \frac{\pi^2 a_3^2 a_4 a_5^5}{10296} + \frac{15091 a_2 a_3^3 a_5^5}{8648640} + \frac{a_4^4 a_5^5}{20592} + \frac{683 a_2^2 a_3 a_5^6}{118800} + \frac{19883 \pi^2 a_2^2 a_3 a_5^6}{38918880} + \frac{10741 a_3^3 a_5^6}{44226000} + \\
 & \frac{1409 \pi^2 a_3^3 a_5^6}{42456960} + \frac{194041 a_2 a_3 a_4 a_5^6}{121621500} + \frac{137 \pi^2 a_2 a_3 a_4 a_5^6}{3335904} + \frac{1409 a_3 a_2^2 a_4^2 a_5^6}{14152320} + \frac{171541 a_2^2 a_5^7}{1702701000} + \\
 & \frac{\pi^2 a_2^2 a_5^7}{555984} + \frac{546901 a_3^2 a_5^7}{40864824000} + \frac{73 \pi^2 a_3^2 a_5^7}{170131104} + \frac{233 a_2 a_4 a_5^7}{9729720} + \frac{73 a_4^2 a_5^7}{56710368} + \frac{107 a_3 a_5^8}{305363520} + \frac{197 a_5^9}{58184837568}
 \end{aligned}$$

10th central moment of 5-term metalog (427 terms)

$$\begin{aligned}
 \mu_{10,5} := & \frac{2555}{33} \pi^{10} a_2^{10} + \frac{16383}{4} \pi^8 a_2^8 a_3^2 + \frac{12775}{44} \pi^{10} a_2^8 a_3^2 + \frac{30535}{8} \pi^6 a_2^6 a_3^4 + \frac{16891}{4} \pi^8 a_2^6 a_3^4 + \\
 & \frac{17885}{88} \pi^{10} a_2^6 a_3^4 + \frac{5831}{32} \pi^4 a_2^4 a_3^6 + \frac{115165}{96} \pi^6 a_2^4 a_3^6 + \frac{139573}{160} \pi^8 a_2^4 a_3^6 + \frac{12775}{352} \pi^{10} a_2^4 a_3^6 + \\
 & \frac{135}{256} \pi^2 a_2^2 a_3^8 + \frac{931}{64} \pi^4 a_2^2 a_3^8 + \frac{23591}{384} \pi^6 a_2^2 a_3^8 + \frac{257683}{6720} \pi^8 a_2^2 a_3^8 + \frac{12775}{8448} \pi^{10} a_2^2 a_3^8 + \frac{a_3^{10}}{11264} + \\
 & \frac{5\pi^2 a_3^{10}}{1024} + \frac{637\pi^4 a_3^{10}}{7680} + \frac{28489\pi^6 a_3^{10}}{96768} + \frac{31369\pi^8 a_3^{10}}{179200} + \frac{2555\pi^{10} a_3^{10}}{371712} + 381\pi^8 a_2^9 a_4 + 6665\pi^6 a_2^7 a_3^2 a_4 + \\
 & 2032\pi^8 a_2^7 a_3^2 a_4 + \frac{17787}{8} \pi^4 a_2^5 a_3^4 a_4 + 5890\pi^6 a_2^5 a_3^4 a_4 + \frac{50673}{40} \pi^8 a_2^5 a_3^4 a_4 + \frac{525}{16} \pi^2 a_2^3 a_3^6 a_4 + \\
 & \frac{931}{2} \pi^4 a_2^3 a_3^6 a_4 + \frac{13485}{16} \pi^6 a_2^3 a_3^6 a_4 + \frac{6477}{40} \pi^8 a_2^3 a_3^6 a_4 + \frac{5}{256} a_2 a_3^8 a_4 + \frac{5}{4} \pi^2 a_2 a_3^8 a_4 + \\
 & \frac{22099\pi^4 a_2 a_3^8 a_4}{1920} + \frac{111259\pi^6 a_2 a_3^8 a_4}{6048} + \frac{465709\pi^8 a_2 a_3^8 a_4}{134400} + 620\pi^6 a_2^8 a_4 + \frac{127}{4} \pi^8 a_2^8 a_4 + \\
 & 3822\pi^4 a_2^6 a_3^2 a_4 + \frac{13175}{4} \pi^6 a_2^6 a_3^2 a_4 + \frac{2667}{20} \pi^8 a_2^6 a_3^2 a_4 + \frac{1575}{4} \pi^2 a_2^4 a_3^4 a_4 + \frac{78645}{32} \pi^4 a_2^4 a_3^4 a_4 + \\
 & \frac{25265}{16} \pi^6 a_2^4 a_3^4 a_4 + \frac{1905}{32} \pi^8 a_2^4 a_3^4 a_4 + \frac{5}{4} a_2^2 a_3^6 a_4 + \frac{2975}{64} \pi^2 a_2^2 a_3^6 a_4 + \frac{39151}{192} \pi^4 a_2^2 a_3^6 a_4 + \\
 & \frac{1481645\pi^6 a_2^2 a_3^6 a_4}{12096} + \frac{889}{192} \pi^8 a_2^2 a_3^6 a_4 + \frac{15a_3^8 a_4}{1024} + \frac{91}{256} \pi^2 a_3^8 a_4 + \frac{6433\pi^4 a_3^8 a_4}{4608} + \frac{7657\pi^6 a_3^8 a_4}{8960} + \\
 & \frac{381\pi^8 a_3^8 a_4}{11264} + 490\pi^4 a_2^7 a_3^3 + \frac{310}{3} \pi^6 a_2^7 a_3^3 + 945\pi^2 a_2^5 a_3^2 a_4 + \frac{4165}{2} \pi^4 a_2^5 a_3^2 a_4 + \frac{4433}{12} \pi^6 a_2^5 a_3^2 a_4 + \\
 & \frac{165}{8} a_2^3 a_3^4 a_4 + \frac{2975}{8} \pi^2 a_2^3 a_3^4 a_4 + \frac{15631}{24} \pi^4 a_2^3 a_3^4 a_4 + \frac{4805}{42} \pi^6 a_2^3 a_3^4 a_4 + \frac{35}{32} a_2 a_3^6 a_4 + \\
 & \frac{2681}{192} \pi^2 a_2 a_3^6 a_4 + \frac{2569}{108} \pi^4 a_2 a_3^6 a_4 + \frac{12803\pi^6 a_2 a_3^6 a_4}{2880} + 210\pi^2 a_2^6 a_4 + \frac{245}{2} \pi^4 a_2^6 a_4 + \frac{31}{8} \pi^6 a_2^6 a_4 +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{165}{2} a_2^4 a_3^2 a_4^4 + \frac{2275}{4} \pi^2 a_2^4 a_3^2 a_4^4 + \frac{30331}{96} \pi^4 a_2^4 a_3^2 a_4^4 + \frac{2325}{224} \pi^6 a_2^4 a_3^2 a_4^4 + \frac{525}{32} a_2^2 a_3^4 a_4^4 + \\
 & \frac{12005}{128} \pi^2 a_2^2 a_3^4 a_4^4 + \frac{32375}{576} \pi^4 a_2^2 a_3^4 a_4^4 + \frac{775}{384} \pi^6 a_2^2 a_3^4 a_4^4 + \frac{91}{512} a_2^6 a_3^4 a_4^4 + \frac{4595}{4608} \pi^2 a_2^6 a_3^4 a_4^4 + \\
 & \frac{1729}{2560} \pi^4 a_2^6 a_3^4 a_4^4 + \frac{155}{5632} \pi^6 a_2^6 a_3^4 a_4^4 + 42 a_2^5 a_4^5 + 70 \pi^2 a_2^5 a_4^5 + \frac{1127}{120} \pi^4 a_2^5 a_4^5 + \frac{105}{2} a_2^3 a_3^2 a_4^5 + \\
 & \frac{1673}{16} \pi^2 a_2^3 a_3^2 a_4^5 + \frac{133}{8} \pi^4 a_2^3 a_3^2 a_4^5 + \frac{441}{128} a_2 a_3^4 a_4^5 + \frac{761}{96} \pi^2 a_2 a_3^4 a_4^5 + \frac{987}{640} \pi^4 a_2 a_3^4 a_4^5 + \frac{35}{2} a_2^4 a_4^6 + \\
 & \frac{49}{6} \pi^2 a_2^4 a_4^6 + \frac{7}{32} \pi^4 a_2^4 a_4^6 + \frac{21}{2} a_2^2 a_3^6 a_4^6 + \frac{4303}{576} \pi^2 a_2^2 a_3^6 a_4^6 + \frac{49}{192} \pi^4 a_2^2 a_3^6 a_4^6 + \frac{919}{4608} a_2^4 a_3^4 a_4^6 + \\
 & \frac{247}{1280} \pi^2 a_2^4 a_3^4 a_4^6 + \frac{49}{5632} \pi^4 a_2^4 a_3^4 a_4^6 + \frac{11}{4} a_2^3 a_4^7 + \frac{11}{28} \pi^2 a_2^3 a_4^7 + \frac{815}{1008} a_2 a_3^2 a_4^7 + \frac{1339}{6720} \pi^2 a_2 a_3^2 a_4^7 + \frac{409}{2016} a_2^2 a_4^7 + \\
 & \frac{5}{768} \pi^2 a_2^2 a_4^8 + \frac{741}{35840} a_2^2 a_4^8 + \frac{15}{11264} \pi^2 a_2^2 a_4^8 + \frac{563}{80640} a_2 a_4^9 + \frac{a_4^{10}}{11264} + 2170 \pi^6 a_2^8 a_3 a_5 + \frac{1143}{4} \pi^8 a_2^8 a_3 a_5 + \\
 & \frac{4655}{2} \pi^4 a_2^6 a_3^3 a_5 + \frac{46655}{12} \pi^6 a_2^6 a_3^3 a_5 + \frac{6223}{12} \pi^8 a_2^6 a_3^3 a_5 + \frac{735}{8} \pi^2 a_2^4 a_3^5 a_5 + \frac{29645}{32} \pi^4 a_2^4 a_3^5 a_5 + \\
 & \frac{18383}{16} \pi^6 a_2^4 a_3^5 a_5 + \frac{4953}{32} \pi^8 a_2^4 a_3^5 a_5 + \frac{5}{32} a_2^7 a_3 a_5 + \frac{525}{64} \pi^2 a_2^7 a_3 a_5 + \frac{53753}{960} \pi^4 a_2^7 a_3 a_5 + \\
 & \frac{111445}{1728} \pi^6 a_2^7 a_3 a_5 + \frac{43307}{4800} \pi^8 a_2^7 a_3 a_5 + \frac{5}{3072} a_2^9 a_5 + \frac{13}{256} \pi^2 a_2^9 a_5 + \frac{20867}{69120} \pi^4 a_2^9 a_5 + \frac{253921}{725760} \pi^6 a_2^9 a_5 + \\
 & \frac{924179}{17740800} \pi^8 a_2^9 a_5 + 3234 \pi^4 a_2^7 a_3 a_4 a_5 + \frac{4495}{3} \pi^6 a_2^7 a_3 a_4 a_5 + \frac{508}{15} \pi^8 a_2^7 a_3 a_4 a_5 + 1155 \pi^2 a_2^5 a_3^3 a_4 a_5 + \\
 & \frac{18669}{4} \pi^4 a_2^5 a_3^3 a_4 a_5 + \frac{7719}{4} \pi^6 a_2^5 a_3^3 a_4 a_5 + \frac{254}{5} \pi^8 a_2^5 a_3^3 a_4 a_5 + \frac{85}{8} a_2^3 a_3^5 a_4 a_5 + \frac{4655}{16} \pi^2 a_2^3 a_3^5 a_4 a_5 +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{7105}{8} \pi^4 a_2^3 a_3^5 a_4 a_5 + \frac{1121363 \pi^6 a_2^3 a_3^5 a_4 a_5}{3024} + \frac{127}{12} \pi^8 a_2^3 a_3^5 a_4 a_5 + \frac{85}{192} a_2^7 a_3 a_4 a_5 + \\
 & \frac{1561}{192} \pi^2 a_2 a_3^7 a_4 a_5 + \frac{1491}{64} \pi^4 a_2 a_3^7 a_4 a_5 + \frac{1877639 \pi^6 a_2 a_3^7 a_4 a_5}{181440} + \frac{127}{396} \pi^8 a_2 a_3^7 a_4 a_5 + \\
 & 1680 \pi^2 a_2^6 a_3 a_4^2 a_5 + \frac{4165}{2} \pi^4 a_2^6 a_3 a_4^2 a_5 + \frac{899}{4} \pi^6 a_2^6 a_3 a_4^2 a_5 + 135 a_2^4 a_3^2 a_4 a_5 + \frac{6475}{4} \pi^2 a_2^4 a_3^2 a_4 a_5 + \\
 & \frac{29253}{16} \pi^4 a_2^4 a_3^2 a_4 a_5 + \frac{221495 \pi^6 a_2^4 a_3^2 a_4 a_5}{1008} + \frac{585}{32} a_2^5 a_3^2 a_4 a_5 + \frac{10633}{64} \pi^2 a_2^5 a_3^2 a_4 a_5 + \\
 & \frac{18767}{96} \pi^4 a_2^5 a_3 a_4^2 a_5 + \frac{58931 \pi^6 a_2^5 a_3 a_4^2 a_5}{2240} + \frac{39}{256} a_3^7 a_4 a_5 + \frac{2981 \pi^2 a_3^7 a_4^2 a_5}{2304} + \frac{57337 \pi^4 a_3^7 a_4^2 a_5}{34560} + \\
 & \frac{225587 \pi^6 a_3^7 a_4^2 a_5}{887040} + 300 a_2^5 a_3 a_4^3 a_5 + 1155 \pi^2 a_2^5 a_3 a_4^3 a_5 + \frac{4655}{12} \pi^4 a_2^5 a_3 a_4^3 a_5 + \frac{62}{7} \pi^6 a_2^5 a_3 a_4^3 a_5 + \\
 & \frac{635}{4} a_2^3 a_3^3 a_4 a_5 + \frac{14105}{24} \pi^2 a_2^3 a_3^3 a_4 a_5 + \frac{50197}{216} \pi^4 a_2^3 a_3^3 a_4 a_5 + \frac{775}{126} \pi^6 a_2^3 a_3^3 a_4 a_5 + \frac{441}{64} a_2 a_3^5 a_4 a_5 + \\
 & \frac{7795}{288} \pi^2 a_2 a_3^5 a_4 a_5 + \frac{109823 \pi^4 a_2 a_3^5 a_4 a_5}{8640} + \frac{155}{396} \pi^6 a_2 a_3^5 a_4 a_5 + \frac{455}{2} a_2^4 a_3 a_4 a_5 + \frac{770}{3} \pi^2 a_2^4 a_3 a_4 a_5 + \\
 & \frac{7721}{288} \pi^4 a_2^4 a_3 a_4 a_5 + \frac{105}{2} a_2^2 a_3^4 a_4 a_5 + \frac{44735}{576} \pi^2 a_2^2 a_3^4 a_4 a_5 + \frac{1981}{192} \pi^4 a_2^2 a_3^4 a_4 a_5 + \frac{2981 a_3^5 a_4^4 a_5}{4608} + \\
 & \frac{8191 \pi^2 a_3^5 a_4 a_5}{6912} + \frac{50939 \pi^4 a_3^5 a_4 a_5}{253440} + \frac{231}{4} a_2^3 a_3^5 a_4 a_5 + \frac{3487}{144} \pi^2 a_2^3 a_3^5 a_4 a_5 + \frac{7}{12} \pi^4 a_2^3 a_3^5 a_4 a_5 + \\
 & \frac{1201}{192} a_2 a_3^3 a_4 a_5 + \frac{6713 \pi^2 a_2 a_3^3 a_4 a_5}{1728} + \frac{49}{396} \pi^4 a_2 a_3^3 a_4 a_5 + \frac{937}{144} a_2 a_3^6 a_4 a_5 + \frac{929}{960} \pi^2 a_2^2 a_3^6 a_4 a_5 + \\
 & \frac{8191 a_3^6 a_4 a_5}{34560} + \frac{7277 \pi^2 a_3^6 a_4 a_5}{126720} + \frac{20063 a_2 a_3^7 a_4 a_5}{60480} + \frac{5}{396} \pi^2 a_2 a_3^7 a_4 a_5 + \frac{7277 a_3 a_4^8 a_5}{1182720} + \\
 & 294 \pi^4 a_2^8 a_3^2 a_5 + \frac{155}{3} \pi^6 a_2^8 a_3^2 a_5 + \frac{127}{60} \pi^8 a_2^8 a_3^2 a_5 + 525 \pi^2 a_2^6 a_3^2 a_5 + \frac{5047}{4} \pi^4 a_2^6 a_3^2 a_5 + \frac{12245}{36} \pi^6 a_2^6 a_3^2 a_5 +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{1397}{180} \pi^8 a_2^6 a_3^2 a_5^2 + \frac{115}{8} a_2^4 a_3^4 a_5^2 + \frac{4375}{16} \pi^2 a_2^4 a_3^4 a_5^2 + \frac{17983}{32} \pi^4 a_2^4 a_3^4 a_5^2 + \frac{507935}{3024} \pi^6 a_2^4 a_3^4 a_5^2 + \\
 & \frac{127}{32} \pi^8 a_2^4 a_3^4 a_5^2 + \frac{275}{192} a_2^2 a_3^6 a_5^2 + \frac{3745}{192} \pi^2 a_2^2 a_3^6 a_5^2 + \frac{11557}{288} \pi^4 a_2^2 a_3^6 a_5^2 + \frac{482081}{36288} \pi^6 a_2^2 a_3^6 a_5^2 + \\
 & \frac{2159}{6336} \pi^8 a_2^2 a_3^6 a_5^2 + \frac{89}{9216} a_2^8 a_3^8 a_5^2 + \frac{15}{128} \pi^2 a_2^8 a_3^8 a_5^2 + \frac{17549}{69120} \pi^4 a_2^8 a_3^8 a_5^2 + \frac{2015}{21384} \pi^6 a_2^8 a_3^8 a_5^2 + \frac{10541}{3953664} \pi^8 a_2^8 a_3^8 a_5^2 + \\
 & 420 \pi^2 a_2^7 a_4 a_5^2 + 245 \pi^4 a_2^7 a_4 a_5^2 + \frac{62}{3} \pi^6 a_2^7 a_4 a_5^2 + 180 a_2^5 a_3^2 a_4 a_5^2 + \frac{2625}{2} \pi^2 a_2^5 a_3^2 a_4 a_5^2 + \\
 & \frac{7497}{8} \pi^4 a_2^5 a_3^2 a_4 a_5^2 + \frac{3565}{42} \pi^6 a_2^5 a_3^2 a_4 a_5^2 + \frac{885}{16} a_2^3 a_3^4 a_4 a_5^2 + \frac{2065}{6} \pi^2 a_2^3 a_3^4 a_4 a_5^2 + \frac{20111}{72} \pi^4 a_2^3 a_3^4 a_4 a_5^2 + \\
 & \frac{43493}{1512} \pi^6 a_2^3 a_3^4 a_4 a_5^2 + \frac{223}{128} a_2^6 a_3^6 a_4 a_5^2 + \frac{3103}{288} \pi^2 a_2^6 a_3^6 a_4 a_5^2 + \frac{172739}{17280} \pi^4 a_2^6 a_3^6 a_4 a_5^2 + \\
 & \frac{131471}{110880} \pi^6 a_2^6 a_3^6 a_4 a_5^2 + 180 a_2^6 a_4 a_5^2 + 315 \pi^2 a_2^6 a_4 a_5^2 + \frac{833}{12} \pi^4 a_2^6 a_4 a_5^2 + \frac{341}{252} \pi^6 a_2^6 a_4 a_5^2 + \\
 & \frac{1245}{4} a_2^4 a_3^2 a_4 a_5^2 + \frac{2835}{4} \pi^2 a_2^4 a_3^2 a_4 a_5^2 + \frac{1547}{8} \pi^4 a_2^4 a_3^2 a_4 a_5^2 + \frac{465}{112} \pi^6 a_2^4 a_3^2 a_4 a_5^2 + \frac{2565}{64} a_2^2 a_3^4 a_4 a_5^2 + \\
 & \frac{6865}{64} \pi^2 a_2^2 a_3^4 a_4 a_5^2 + \frac{2303}{64} \pi^4 a_2^2 a_3^4 a_4 a_5^2 + \frac{13175}{14784} \pi^6 a_2^2 a_3^4 a_4 a_5^2 + \frac{45}{128} a_3^6 a_4^2 a_5^2 + \frac{2507}{2304} \pi^2 a_3^6 a_4^2 a_5^2 + \\
 & \frac{3185}{7128} \pi^4 a_3^6 a_4^2 a_5^2 + \frac{12865}{988416} \pi^6 a_3^6 a_4^2 a_5^2 + 140 a_2^5 a_4 a_5^3 + \frac{1085}{12} \pi^2 a_2^5 a_4 a_5^3 + 7 \pi^4 a_2^5 a_4 a_5^3 + \\
 & \frac{7175}{48} a_2^3 a_3^2 a_4 a_5^2 + \frac{845}{6} \pi^2 a_2^3 a_3^2 a_4 a_5^2 + \frac{371}{27} \pi^4 a_2^3 a_3^2 a_4 a_5^2 + \frac{1255}{144} a_2^4 a_3^3 a_4 a_5^2 + \frac{18911}{1728} \pi^2 a_2^4 a_3^3 a_4 a_5^2 + \\
 & \frac{721}{528} \pi^4 a_2^4 a_3^3 a_4 a_5^2 + \frac{175}{4} a_2^4 a_4 a_5^2 + \frac{1675}{144} \pi^2 a_2^4 a_4 a_5^2 + \frac{7}{32} \pi^4 a_2^4 a_4 a_5^2 + \frac{5455}{192} a_2^2 a_3^4 a_4 a_5^2 + \\
 & \frac{6689}{576} \pi^2 a_2^2 a_3^4 a_4 a_5^2 + \frac{595}{2112} a_2^2 a_3^4 a_4 a_5^2 + \frac{2507}{4608} \pi^4 a_2^2 a_3^4 a_4 a_5^2 + \frac{2275}{7128} \pi^2 a_2^2 a_3^4 a_4 a_5^2 + \frac{20335}{1976832} \pi^4 a_2^2 a_3^4 a_4 a_5^2 +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{163}{24} \frac{a_2^3 a_4^5 a_5^2}{a_2^2 a_4 a_5} + \frac{239}{360} \pi^2 \frac{a_2^3 a_4^5 a_5^2}{a_2^2 a_4 a_5} + \frac{2629 a_2 a_3^5 a_4^5 a_5^2}{1152} + \frac{1939 \pi^2 a_2 a_3^5 a_4^5 a_5^2}{5280} + \frac{4711 a_2^2 a_4^6 a_5^2}{8640} + \\
 & \frac{85 \pi^2 a_2^2 a_4^6 a_5^2}{6336} + \frac{7128}{988416} \frac{455 a_2^2 a_3^6 a_4^6 a_5^2}{a_2^2 a_3^6 a_4^6 a_5^2} + \frac{2905 \pi^2 a_2^2 a_3^6 a_4^6 a_5^2}{9240} + \frac{197 a_2 a_4^7 a_5^2}{1317888} + \frac{415 a_4^8 a_5^2}{45 a_2^6 a_3 a_5} + \\
 & \frac{175 \pi^2 a_2^6 a_3 a_5^3}{24} + \frac{2009}{126} \pi^4 \frac{a_2^6 a_3 a_5^3}{a_2^6 a_3 a_5^3} + \frac{775}{126} \pi^6 \frac{a_2^6 a_3 a_5^3}{a_2^6 a_3 a_5^3} + \frac{65}{2} \frac{a_4^3 a_3^3}{a_2^4 a_3^3 a_5} + \frac{6335}{48} \pi^2 \frac{a_4^3 a_3^3}{a_2^4 a_3^3 a_5} + \\
 & \frac{65779}{864} \pi^4 \frac{a_4^3 a_3^3}{a_2^4 a_3^3 a_5} + \frac{28613 \pi^6 a_2^4 a_3^3 a_5^3}{4536} + \frac{351}{128} \frac{a_2^5 a_3^3}{a_2^2 a_3^5 a_5} + \frac{1741}{144} \pi^2 \frac{a_2^5 a_3^3}{a_2^2 a_3^5 a_5} + \frac{47621 \pi^4 a_2^5 a_3^3 a_5^3}{5760} + \\
 & \frac{787121 \pi^6 a_2^5 a_3^3 a_5^3}{997920} + \frac{589 a_3^7 a_5^3}{32256} + \frac{203 \pi^2 a_3^7 a_5^3}{2304} + \frac{161917 \pi^4 a_3^7 a_5^3}{2280960} + \frac{2866973 \pi^6 a_3^7 a_5^3}{363242880} + \\
 & \frac{135 a_2^5 a_3 a_4 a_5^3}{4} + \frac{735}{4} \pi^2 \frac{a_2^5 a_3 a_4 a_5^3}{a_2^5 a_3 a_4 a_5^3} + \frac{679}{18} \pi^4 \frac{a_2^5 a_3 a_4 a_5^3}{a_2^5 a_3 a_4 a_5^3} + \frac{124}{189} \pi^6 \frac{a_2^5 a_3 a_4 a_5^3}{a_2^5 a_3 a_4 a_5^3} + \frac{835}{16} \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \\
 & \frac{6775}{72} \pi^2 \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \frac{2555}{108} \pi^4 \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \frac{2015 \pi^6 a_2^3 a_3 a_4 a_5^3}{4158} + \frac{1121}{576} \frac{a_2^5 a_3 a_4 a_5^3}{a_2^5 a_3 a_4 a_5^3} + \\
 & \frac{7469 \pi^2 a_2 a_3^5 a_4 a_5^3}{1728} + \frac{191303 \pi^4 a_2 a_3^5 a_4 a_5^3}{142560} + \frac{775 \pi^6 a_2 a_3^5 a_4 a_5^3}{24024} + \frac{385}{4} \frac{a_2^4 a_3 a_4 a_5^3}{a_2^4 a_3 a_4 a_5^3} + \\
 & \frac{2905}{48} \pi^2 \frac{a_2^4 a_3 a_4 a_5^3}{a_2^4 a_3 a_4 a_5^3} + \frac{329}{72} \pi^4 \frac{a_2^4 a_3 a_4 a_5^3}{a_2^4 a_3 a_4 a_5^3} + \frac{12695}{576} \frac{a_2^3 a_3^2 a_4 a_5^3}{a_2^3 a_3^2 a_4 a_5^3} + \frac{16621}{864} \pi^2 \frac{a_2^3 a_3^2 a_4 a_5^3}{a_2^3 a_3^2 a_4 a_5^3} + \\
 & \frac{26327 \pi^4 a_2 a_3^2 a_4 a_5^3}{14256} + \frac{203}{768} \frac{a_3^5 a_2^2 a_4 a_5^3}{a_3^5 a_2^2 a_4 a_5^3} + \frac{23131 \pi^2 a_3^5 a_2^2 a_4 a_5^3}{76032} + \frac{92483 \pi^4 a_3^5 a_2^2 a_4 a_5^3}{2471040} + \frac{1355}{48} \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \\
 & \frac{863}{108} \pi^2 \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \frac{91}{594} \pi^4 \frac{a_2^3 a_3 a_4 a_5^3}{a_2^3 a_3 a_4 a_5^3} + \frac{5647 a_2 a_3^3 a_4 a_5^3}{1728} + \frac{6439 \pi^2 a_2 a_3^3 a_4 a_5^3}{4752} + \frac{175 \pi^4 a_2 a_3^3 a_4 a_5^3}{5148} + \\
 & \frac{4423 a_2^2 a_3 a_4 a_5^3}{1152} + \frac{4073 \pi^2 a_2^2 a_3 a_4 a_5^3}{9504} + \frac{23131 a_3^3 a_4 a_5^3}{152064} + \frac{92483 \pi^2 a_3^3 a_4 a_5^3}{3459456} + \frac{2261 a_2 a_3 a_4^5 a_5^3}{9504} +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{25 \pi^2 a_2 a_3 a_4 a_5^5}{3432} + \frac{92 483 a_3 a_4 a_5^6}{17 297 280} + \frac{15 a_2^6 a_5^4}{24} + \frac{175 \pi^2 a_2^6 a_5^4}{72} + \frac{77 \pi^4 a_2^6 a_5^4}{72} + \frac{31 \pi^6 a_2^6 a_5^4}{1512} + \\
 & \frac{425 a_2^4 a_2^4 a_3 a_5}{32} + \frac{1585 \pi^2 a_2^4 a_3 a_5^4}{96} + \frac{1393 \pi^4 a_2^4 a_3 a_5^4}{432} + \frac{11 315 \pi^6 a_2^4 a_3 a_5^4}{199 584} + \frac{1835 a_2^2 a_3^4 a_5^4}{1152} + \\
 & \frac{737 \pi^2 a_2^2 a_3 a_5^4}{288} + \frac{70 651 \pi^4 a_2^2 a_3 a_5^4}{114 048} + \frac{43 555 \pi^6 a_2^2 a_3 a_5^4}{3 459 456} + \frac{917 a_3^6 a_5^4}{69 120} + \frac{1003 \pi^2 a_3^6 a_5^4}{38 016} + \\
 & \frac{239 623 \pi^4 a_3^6 a_5^4}{30 326 400} + \frac{23 591 \pi^6 a_3^6 a_5^4}{124 540 416} + \frac{91 a_2^5 a_4 a_5^4}{8} + \frac{329 \pi^4 a_2^5 a_4 a_5^4}{1080} + \frac{935 a_2^3 a_3^2 a_4 a_5^4}{72} + \\
 & \frac{1175 \pi^2 a_2^3 a_3^2 a_4 a_5^4}{144} + \frac{1477 \pi^4 a_2^3 a_3^2 a_4 a_5^4}{2376} + \frac{671 a_2^4 a_3 a_4 a_5^4}{864} + \frac{18 763 \pi^2 a_2^4 a_3 a_4 a_5^4}{28 512} + \\
 & \frac{284 045 \pi^4 a_2^4 a_3 a_4 a_5^4}{4 447 872} + \frac{535 a_2^4 a_4 a_5^4}{96} + \frac{163 \pi^2 a_2^4 a_4 a_5^4}{144} + \frac{511 \pi^4 a_2^4 a_4 a_5^4}{28 512} + \frac{377 a_2^2 a_3^2 a_4 a_5^4}{96} + \\
 & \frac{22 499 \pi^2 a_2^2 a_3^2 a_4 a_5^4}{19 008} + \frac{1967 \pi^4 a_2^2 a_3^2 a_4 a_5^4}{82 368} + \frac{1003 a_3^4 a_2^2 a_4 a_5^4}{12 672} + \frac{239 623 \pi^2 a_3^4 a_2^2 a_4 a_5^4}{7 076 160} + \frac{5327 \pi^4 a_3^4 a_2^2 a_4 a_5^4}{5 930 496} + \\
 & \frac{89 a_2^3 a_4 a_5^4}{72} + \frac{353 \pi^2 a_2^3 a_3 a_4 a_5^4}{3564} + \frac{1427 a_2 a_2^3 a_3 a_4 a_5^4}{3168} + \frac{295 847 \pi^2 a_2 a_2^3 a_3 a_4 a_5^4}{5 189 184} + \frac{5101 a_2^2 a_4 a_5^4}{38 016} + \\
 & \frac{1405 \pi^2 a_2^2 a_4 a_5^4}{494 208} + \frac{239 623 a_2^4 a_3 a_4 a_5^4}{14 152 320} + \frac{3805 \pi^2 a_2^4 a_3 a_4 a_5^4}{5 930 496} + \frac{354 857 a_2 a_2^4 a_3 a_4 a_5^4}{51 891 840} + \frac{761 a_4^6 a_5^4}{5 930 496} + \\
 & \frac{419 a_2^4 a_3 a_5^5}{288} + \frac{241 \pi^2 a_2^4 a_3 a_5^5}{360} + \frac{133 \pi^4 a_2^4 a_3 a_5^5}{3168} + \frac{3001 a_2^2 a_3^3 a_5^5}{8640} + \frac{63 091 \pi^2 a_2^2 a_3^3 a_5^5}{285 120} + \\
 & \frac{159 \pi^4 a_2^2 a_3^3 a_5^5}{9152} + \frac{4889 a_3^5 a_5^5}{1 140 480} + \frac{1 872 859 \pi^2 a_3^5 a_5^5}{518 918 400} + \frac{96 209 \pi^4 a_3^5 a_5^5}{266 872 320} + \frac{77 a_2^3 a_3 a_4 a_5^5}{72} + \\
 & \frac{1907 \pi^2 a_2^3 a_3 a_4 a_5^5}{7920} + \frac{553 \pi^4 a_2^3 a_3 a_4 a_5^5}{138 996} + \frac{12 467 a_2 a_2^3 a_3 a_4 a_5^5}{95 040} + \frac{16 440 139 \pi^2 a_2 a_2^3 a_3 a_4 a_5^5}{389 188 800} +
 \end{aligned}$$

10th central moment of 5-term metalog (continued)

$$\begin{aligned}
 & \frac{7 \pi^4 a_2 a_3^3 a_4 a_5^5}{7722} + \frac{817 a_2^2 a_3 a_4 a_5^5}{3168} + \frac{124 759 \pi^2 a_2^2 a_3 a_4 a_5^5}{5189184} + \frac{1 872 859 a_3^3 a_4 a_5^5}{172 972 800} + \frac{96 209 \pi^2 a_3^3 a_4 a_5^5}{62 270 208} + \\
 & \frac{9 503 057 a_2 a_3 a_4 a_5^5}{389 188 800} + \frac{5 \pi^2 a_2 a_3 a_4 a_5^5}{7722} + \frac{96 209 a_3 a_4 a_5^5}{124 540 416} + \frac{191 a_2^4 a_5^6}{4320} + \frac{53 \pi^2 a_2^4 a_5^6}{7128} + \frac{371 \pi^4 a_2^4 a_5^6}{3 335 904} + \\
 & \frac{623 a_2^2 a_3^2 a_5^6}{19008} + \frac{242 387 \pi^2 a_2^2 a_3^2 a_5^6}{29 937 600} + \frac{959 \pi^4 a_2^2 a_3^2 a_5^6}{6 671 808} + \frac{1 076 363 a_3^4 a_5^6}{1 556 755 200} + \frac{11 231 \pi^2 a_3^4 a_5^6}{47 174 400} + \\
 & \frac{4991 \pi^4 a_3^4 a_5^6}{907 365 888} + \frac{79 a_2^3 a_4 a_5^6}{2970} + \frac{3623 \pi^2 a_2^3 a_4 a_5^6}{1 945 944} + \frac{3 983 627 a_2 a_3^2 a_4 a_5^6}{389 188 800} + \frac{12 823 \pi^2 a_2 a_3^2 a_4 a_5^6}{11 675 664} + \\
 & \frac{1 047 889 a_2^2 a_4 a_5^6}{194 594 400} + \frac{685 \pi^2 a_2^2 a_4 a_5^6}{6 671 808} + \frac{11 231 a_3^2 a_4 a_5^6}{15 724 800} + \frac{3565 \pi^2 a_3^2 a_4 a_5^6}{151 227 648} + \frac{10 147 a_2 a_3^3 a_5^6}{23 351 328} + \\
 & \frac{3565 a_4^6}{302 455 296} + \frac{734 273 a_2^2 a_3 a_5^7}{544 864 320} + \frac{431 \pi^2 a_2^2 a_3 a_5^7}{3 891 888} + \frac{386 887 a_3^3 a_5^7}{6 538 371 840} + \frac{5755 \pi^2 a_3^3 a_5^7}{793 945 152} + \\
 & \frac{368 213 a_2 a_3 a_4 a_5^7}{1 021 620 600} + \frac{365 \pi^2 a_2 a_3 a_4 a_5^7}{42 532 776} + \frac{5755 a_3 a_4 a_5^7}{264 648 384} + \frac{45 571 a_2^8 a_5^8}{2 335 132 800} + \frac{475 \pi^2 a_2^8 a_5^8}{1 361 048 832} + \\
 & \frac{1 633 607 a_3^2 a_5^8}{606 285 388 800} + \frac{24 785 \pi^2 a_3^2 a_5^8}{310 319 133 696} + \frac{14 453 a_2 a_4 a_5^8}{3 175 780 608} + \frac{24 785 a_4^8 a_5^8}{103 439 711 232} + \frac{24 383 a_3 a_5^9}{402 265 543 680} + \frac{161 a_5^{10}}{310 319 133 696}
 \end{aligned}$$